Relativity: pset 3 Due 3/6/18

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1. Lately, CERN upgraded their super collider. They increased the energy of the accelerated protons from 3.5 TeV to 7 TeV. We would like to know how it affected the velocity of the protons. Invariant mass of the proton is 938 $\frac{\text{MeV}}{c^2}$.

2. A particle of mass M and 4-momentum P decays into two particles of masses m_1 and m_2.

   (a) Use the conservation of energy and momentum in the form $p_2 = P - p_1$ to show that the total energy of the first particle in the rest frame of the decaying particle is

   $$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

   and that E_2 is obtained by interchanging m_1 and m_2.

   (b) Show that the kinetic energy T_i of the ith particle in the same frame is

   $$T_i = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

   (c) The charged pi-meson ($M = 139.6$ MeV) decays into a mu-meson ($m_1 = 105.7$ MeV) and a neutrino ($m_2 = 0$). Calculate the kinetic energies of the mu-meson and the neutrino in the pi-meson's rest frame. It entered importantly in the discovery of the pi-meson in photographic emulsions by Powell and coworkers in 1947.

3. Two equal masses are connected by a massless string with tension T. (By 'massless', we mean that it has no mass in its unstretched, zero-length state.) The masses are constrained to move with speed v along parallel lines. The constraints are then removed, and the masses are drawn together. They collide and make one blob which continues to move to the right. Is the following reasoning correct? If your answer is 'no', then state what is invalid about whichever sentence(s).
   'The forces on the masses point in the y-direction. Therefore, there is no change in momentum in the x-direction. But the mass of the resulting blob is greater than the sum of the initial masses (because they collided with some relative speed). Therefore, the speed of the resulting blob must be less than v (to keep p_x constant), so the whole apparatus slows down in the x-direction.' (*Hint: This is not correct.*)

4. A long cart moves at relativistic speed v. Sand is dropped into the cart at a rate $\frac{dm}{dt} = \sigma$ in the ground frame. Assume that you stand on the ground next to where the sand falls in, and you push on the cart to keep it moving at constant speed v. What is the force between your feet and the ground? Calculate this force in both the ground frame (your frame) and the cart frame, and show that the results are equal (as should be the case for longitudinal forces).

5. If a system of mass M decays or transforms at rest into a number of particles, the sum of whose masses is less than M by an amount ΔM,

   (a) show that the maximum kinetic energy of the ith particle (mass m_i) is

   $$(T_i)_{\max} = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

   (b) determine the maximum kinetic energies in MeV and also the ratios to ΔM for each of the particles in the following decays or transformations of particles at rest:

   $$\mu \to e + \nu + \overline{\nu}$$

   $$K^+ \to \pi^+ + \pi^- + \pi^+$$

$$K^{\pm} \to e^{\pm} + \pi^0 + \nu$$
$$K^{\pm} \to \mu^{\pm} + \pi^0 + \nu$$
$$p + \overline{p} \to 2\pi^+ + 2\pi^- + \pi^0$$
$$p + \overline{p} \to K^+ + K^- + 3\pi^0$$

6. A rod moves on top of a wire grid that is stretched between two buildings. The rest length of the rod is equal to the spacing between the wires in the grid. In the rest frame of the grid, the rod's Lorentz contraction makes it smaller than the grid spacing; observers in this frame expect it to fall through the grid. In the rest frame of the rod, in contrast, the grid spacing is contracted and it should pass over the grid without any difficulty. These two predictions are contradictory. Which is correct?