Relativity: pset 2 Due 8/2/18
Lecturer: Dimitrios Kidonakis

1. A man walks very fast over a rectangular grid, of the type used in some bridge roadways. The rest length of the walker's foot is equal to the spacing between the grid elements. In the rest frame of the grid, his Lorentz contraction makes him narrower than the grid spacing; observers in that frame expect him to fall in. In the rest frame of the walker, in contrast, the grid spacing is contracted and he should pass over the grid without any difficulty. These two predictions are contradictory. Which is correct?

2. Assume that a rocket ship leaves the earth in the year 2010. One of a set of twins born in 2080 remains on earth; the other rides in the rocket. The rocket ship is so constructed that it has an acceleration g in its own rest frame (this makes the occupants feel at home). It accelerates in a straight-line path for 5 years (by its own clocks), decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. The twin in the rocket is 40 years old.

   (a) What is the year on Earth?

   (b) How far from Earth did the space ship travel?

3. A particle of mass M and 4-momentum P decays into two particles of masses m_1 and m_2.

   (a) Use the conservation of energy and momentum in the form $p_2 = P - p_1$ to show that the total energy of the first particle in the rest frame of the decaying particle is
   $$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$
   and that E_2 is obtained by interchanging m_1 and m_2.

   (b) Show that the kinetic energy T_i of the ith particle in the same frame is
   $$T_i = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

   (c) The charged pi-meson ($M = 139.6$ MeV) decays into a mu-meson ($m_1 = 105.7$ MeV) and a neutrino ($m_2 = 0$). Calculate the kinetic energies of the mu-meson and the neutrino in the pi-meson's rest frame. It entered importantly in the discovery of the pi-meson in photographic emulsions by Powell and coworkers in 1947.

4. A long cart moves at relativistic speed v. Sand is dropped into the cart at a rate $\frac{dm}{dt} = \sigma$ in the ground frame. Assume that you stand on the ground next to where the sand falls in, and you push on the cart to keep it moving at constant speed v. What is the force between your feet and the ground? Calculate this force in both the ground frame (your frame) and the cart frame, and show that the results are equal (as should be the case for longitudinal forces).

5. If a system of mass M decays or transforms at rest into a number of particles, the sum of whose masses is less than M by an amount ΔM,

   (a) show that the maximum kinetic energy of the ith particle (mass m_i) is
   $$(T_i)_{\max} = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

(b) determine the maximum kinetic energies in MeV and also the ratios to ΔM for each of the particles in the following decays or transformations of particles at rest:

$$\mu \to e + \nu + \overline{\nu}$$

$$K^+ \to \pi^+ + \pi^- + \pi^+$$

$$K^\pm \to e^\pm + \pi^0 + \nu$$

$$K^\pm \to \mu^\pm + \pi^0 + \nu$$

$$p + \overline{p} \to 2\pi^+ + 2\pi^- + \pi^0$$

$$p + \overline{p} \to K^+ + K^- + 3\pi^0$$