Relativity Lecture 1: Basic Effects

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Table of Contents

Table of Contents

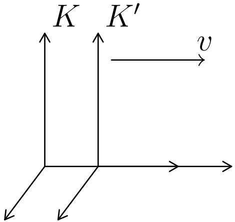
Introduction

- Welcome to Ph 504, where we will learn special relativity and maybe some general relativity too
- Special relativity will take 3-4 weeks
- Today we will do basic relativistic effects
- Ph 504 is a combination of *The Classical Theory of Fields*, Morin's *Introduction to Classical Mechanics*, and MIT's 8.033.

Table of Contents

Newtonian Mechanics

- Two theories at the end of the 19th century: classical mechanics and electromagnetism
- Classical mechanics: time and space are absolute
- In describing the motion of bodies, we need to assign a system of coordinates to make measurements. Some are simpler than others.
- An inertial frame K relative to an event *with no forces acting on it* will measure the event as having constant velocity.



- Another frame K' inertial to K is also inertial to the event
- The frame in which the event is stationary is called the *proper frame*

Newtonian Mechanics

- Non-inertial frames will see the box accelerating due to a (pseudo-)force.
- So inertial frames are particularly nice and result in simpler equations
- How do the measured equations change as we transform from one inertial frame to another?
- Galilean transformation

$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t.
\end{aligned}
$$

Electromagnetism

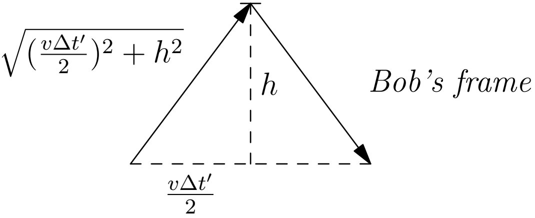
- Maxwell's equations are not invariant under the Galilean transformation; the equations are different and not so simple under Galilean transformation
- Maxwell's equations imply that light travels at a constant velocity c, but the Galilean transformations would imply that this is not the case in other frames
- The special frame with c was called the ether frame
- Michelson-Morley experiment confirms the constancy of the speed of light; furthermore, c is the upper bound on the velocity of any physical object
- Lorentz derived the Lorentz transformations in accordance with Maxwell's equations, as well as local time, but thought it was distinct from real time (ether)
- Poincare deduced the principle of relativity, lack of simultaneity, and more
- Einstein created a firm theory of relativity, grounded in two axioms. Einstein discarded the ether.

Relativity

- No preferred frame
- Saying that something is moving does not make sense, since some frames measure movement while others do not
- You can place the origin of your coordinates anywhere; space is homogeneous and isotropic
- 1st postulate of relativity: The laws of physics are the same in all inertial frames of reference.
- 2nd postulate of relativity: The speed of light in vacuum has the same value c in all inertial frames of reference.

Table of Contents

Train Gedankenexperiment



- Alice's speed of light:

$$c = \frac{2h}{\Delta t}$$

- Bob's speed of light:

$$c = \frac{2\sqrt{(\frac{v\Delta t'}{2})^2 + h^2}}{\Delta t'} = \frac{\sqrt{(v\Delta t')^2 + (2h)^2}}{\Delta t'} = \frac{\sqrt{(v\Delta t')^2 + (c\Delta t)^2}}{\Delta t'}$$

$$\implies \Delta t'^2 \frac{c^2 - v^2}{c^2} = \Delta t^2 \implies \boxed{\Delta t' = \gamma \Delta t}.$$

Time Dilation

- Since $\gamma > 1$, Bob sees that *all natural processes* within the train have been slowed by a factor of γ as compared with Alice's frame and in particular, he sees his clock as ticking quicker than Alice's clock
- For $v << c$,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \to 1,$$

  in which case $\Delta t \approx \Delta t'$
- For $v \approx c$,

$$\gamma \to \infty,$$

  which means Bob sees that all processes in the train have stopped entirely.

Time Dilation

- In the situation above, you could argue that "everything is relativity" and Alice sees Bob moving at speed v, so his clock is slower than her's by a factor of γ, so $\Delta t = \gamma \Delta t'$, but clearly both equations cannot be true.
- Asymmetry: Yes, Alice sees Bob's clock as slower, but how Alice and Bob view each other is irrelevant because we are considering an event within Alice's frame
- Alice's frame is proper
- $\Delta t = \gamma \Delta t_p$.

Time Dilation: Token Muon Example

- A muon is created in the atmosphere of Earth as cosmic rays collide with air molecules, 50km from the surface.
- Muons have a lifetime of $2 \cdot 10^{-6}$ seconds and move at $v = 0.99998c$.
- Does the muon reach the surface of the Earth?
- $d = rt$, right? So $d = (2 \cdot 10^{-6}s)(0.99998 \cdot 299,792,458m/s) = 599.57m$, so the muon does not reach the surface
- However, muons are experimentally detected on the surface of the Earth, at the rate expected that the muons have enough time to reach the surface
- Time is dilated by a factor of $\gamma = \frac{1}{\sqrt{1-0.99998^2}} = 158.11$, so really $d = 158.11 \cdot 599.57 = 95km$.

Length Contraction

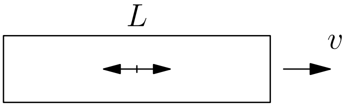
- Now that we got time dilation, length contraction is easy
- A stick of length L flies over a point B on the ground at velocity v.
- Consider the time interval for the stick to clear the point.
- Stick's frame: $L = vt$
- Time interval of B in stick's frame: $\frac{L}{v\gamma}$
- Length in B's frame: $\frac{L}{v\gamma}v = \frac{L}{\gamma}$
- The stick's proper length is L, so $\boxed{L_p = \gamma L}$

Muons Again

- Earlier we showed the calculation in the Earth's reference frame
- From the muon's reference frame, it has to reach the Earth before $2 \cdot 10^{-6}$ seconds elapses
- The distance to the surface is smaller by a factor of γ
- $\frac{d}{\gamma} = vt \implies d = v(\gamma t)$, so it's really the same thing
- Length contraction and time dilation are complementary to each other; in one frame, one experiences length contraction, and in the other, time dilation. So we see that there is a symmetry between time and length

Loss of Simultaneity

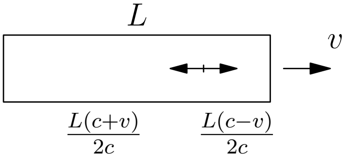
- You set up clocks at opposite sides of the train. To synchronize them, you send a beam of light to each side from the center of the train.



- This procedure works in your reference frame, but to an outside observer, the back end of the train catches up with the the light going to the left, and the front end moves away from the light going to the left; so the back clock is synchronized ahead of the front clock

Loss of Simultaneity

- Now place the clock so that the clocks are synchronized according to the outside observer.



- In the frame of the train, the light going to the left travels an extra distance of

$$\frac{L(c+v)}{2c} - \frac{L(c-v)}{2c} = \frac{Lv}{c},$$

so, by $\frac{Lv}{c} = ct$, the extra time is

$$\boxed{\frac{Lv}{c^2}}$$

The Barn Paradox

- Consider a rod moving through a barn at speed v. The barn has doors at the left and right. A stationary farmer closes the doors at the same time in order to trap the rod.
- In the rod's frame, the barn's length decreases, and so it cannot be trapped
- In the farmer's frame, the length of the rod is contracted, and it easily fits inside the barn
- Having the rod trapped inside the barn is something everyone should agree on.
- The resolution is that, although the doors close at the same time in the farmer's frame, they do not close simultaneously in the rod's frame; in fact, we know that the left door closes at a time $\frac{vL_b}{c^2}$ after the right door. As the rod moves through the barn (in the rod's frame), the right door closes and the left door remains open. The right door opens allowing the rod to exit, and the left door closes.

Summary of Effects

- A train is passing you on a station. I am inside.
- You see that time itself has slowed down in the train by γ; I look younger.
- You see that the train is shorter than usual ($\frac{L_t}{\gamma}$).
- You find that the clock at the end of the train is ahead by time $\frac{L_t v}{c^2}$ than the front clock.
- I look at you and you look younger than me.
- I see the train shorter by $\frac{1}{\gamma}$, and it looks like the clock at the far part of the station is ahead of the clock in the near part of the station, by $\frac{vL_s}{c^2}$.